WRITTEN ASSIGNMENT #3 - SOLUTION

1. (4 points) Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ whose graph has horizontal tangents at the points $(-2, 6)$ and $(2, 0)$.

Solution: Note that the derivative of $f(x)$ with respect to x is

$$f'(x) = 3ax^2 + 2bx + c,$$

since we have 4 unknown parameters then we need 4 equation 2 of which are coming from the $f(x)$ and the other 2 from the $f'(x)$:

$$\begin{cases} a(-2)^3 + b(-2)^2 + c(-2) + d = 6 \\ a(2)^3 + b(2)^2 + c(2) + d = 0 \\ 3a(-2)^2 + 2b(-2) + c = 0 \\ 3a(2)^2 + 2b(2) + c = 0 \end{cases} = \begin{cases} -8a + 4b - 2c + d = 6 \\ 8a + 4b + 2c + d = 0 \\ 12a - 4b + c = 0 \\ 12a + 4b + c = 0 \end{cases}.$$

Using your favorite method, we can solve the above system of equations and get that

$$a = \frac{3}{16},\ b = 0,\ c = -\frac{9}{4},\ \text{and } d = 3.$$

And the cubic function $f(x)$ has the following form:

$$\boxed{f(x) = \frac{3}{16}x^3 - \frac{9}{4}x + 3}.$$

2. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

where μ is a constant called the coefficient of friction.

(a) (2 points) Find the rate of change of F with respect to θ.

Solution: Using quotient rule, we get

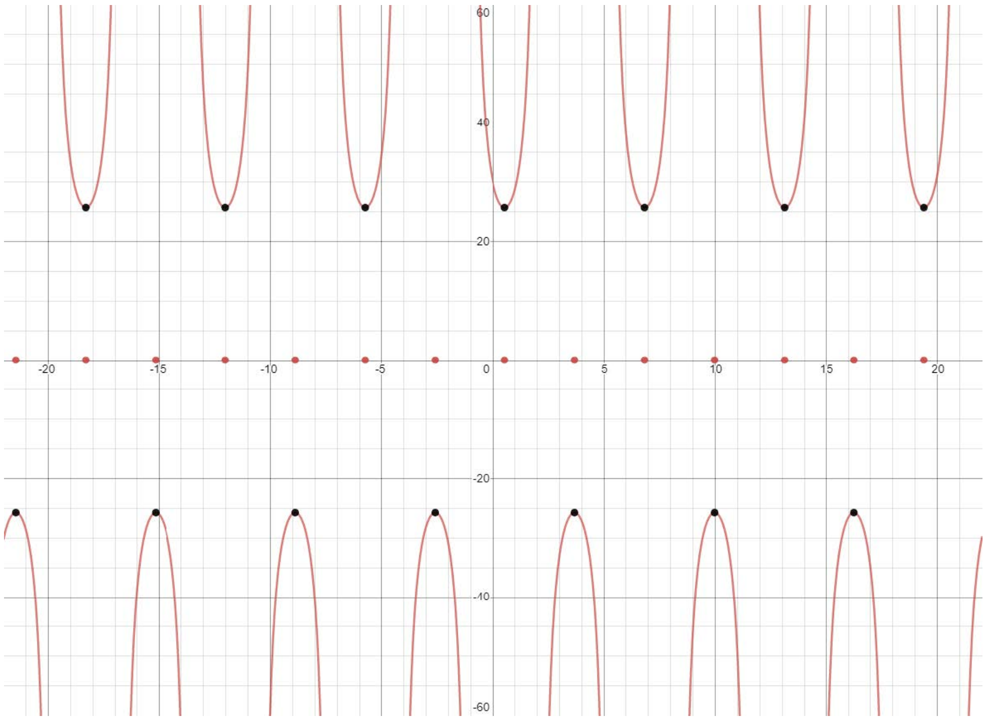
$$\frac{dF}{d\theta} = \frac{0(\mu\sin\theta + \cos\theta) - \mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2} = \boxed{\frac{\mu W(\sin\theta - \mu\cos\theta)}{(\mu\sin\theta + \cos\theta)^2}}$$

(b) (1 point) When is this rate of change equal to 0?

Solution: The rate of change of F equals to 0, when either $\mu = 0$ or $W = 0$ or $\sin\theta = \mu\cos\theta$, i.e. $\mu = \tan\theta$ or $\theta = \tan^{-1}(\mu) + \pi k$ for any $k \in \mathbb{Z}$.

(c) (1 point) If $W = 50$ lb and $\mu = 0.6$, draw the graph of F as a function of θ and use it to locate the value of θ for which $\frac{dF}{d\theta} = 0$. Is the value consistent with your answer to part (b)?

Solution:



All the points at which $\frac{dF}{d\theta} = 0$ are located at the points that we derived in part (b): $\theta = \tan^{-1}(0.6) + \pi k$ for any $k \in \mathbb{Z}$.